ELECTROSTATICS PART 3

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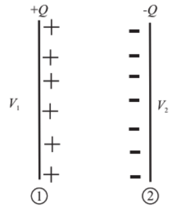
Capacitance:

The work done in moving a unit positive charge from plate 2 to 1 will be proportional to Q hence, V α Q . Thus Q/V is a constant

Thus C=Q/V, where C is called capacitance of capacitor

SI Unit: Farad (F) Dimensions: $[M^{-1} L^{-2} T^4 A^2]$

Smaller units: $1\mu F=10^{-6}$ F, $1nF=10^{-9}$F, $1pF=10^{-12}$ F

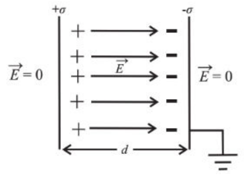


Expression for Capacitance of capacitor:

Case 1: Without dielectric

A=Area of the plates which are parallel to each other and separated by distance d.

Let +Q be on the left plate. The right plate will induce -Q on the inner side and +Q on the outerside, which would be neutralized once the plate is earthed. Thus, for outer plates net E=0.



For inner sides of the plates electric field will be from + to – and given by

$$E = \frac{\sigma}{2\varepsilon o} + \frac{\sigma}{2\varepsilon o} = \frac{\sigma}{\varepsilon o} = \frac{Q}{A\varepsilon o}$$

where σ=surface charge density.

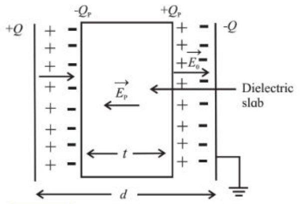
Since E = V/d.

$$Thus, \frac{V}{d} = \frac{Q}{A\varepsilon o} \qquad Hence\ C = \frac{Q}{V} = \frac{\varepsilon o A}{d}$$

Case 2: With dielectric medium

A=Area of the plates which are parallel to each other and separated by distance d and containing a dielectric medium of dielectric constant k and having thickness t (t<d).

σ = surface charge density of the plate = Q/A and Eo is the electric field before the dielectric is introduced (or in the region where the dielectric isn't there).



$$Thus\ Eo = \frac{\sigma}{\varepsilon o} = \frac{Q}{A\varepsilon o}$$

As seen Q induces –Qp on the dielectric on the closer side and +Qp on the far side of the dielectric. Thus creating and electric field in the dielectric Ep in the opposite direction of Eo.

$$Ep = \frac{\sigma_p}{\varepsilon o} = \frac{Q_p}{A\varepsilon o}$$

$$Net\ E = Eo - Ep = \frac{Eo}{k} = \frac{Q}{kA\varepsilon o}$$

$$Thus, V = Eo(d-t) + E.t = Eo(d-t) + \frac{Eo}{k}.t = Eo\left[d - t + \frac{t}{k}\right]$$

$$V = \frac{Q}{A\varepsilon o}\left[d - t + \frac{t}{k}\right]$$

$$C = \frac{Q}{V} = \frac{\varepsilon o A}{\left[d - t + \frac{t}{k}\right]}$$

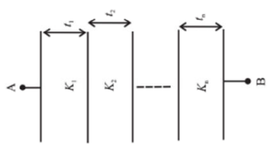
Special Case 1: t=d (the dielectric fills the full space)

$$Thus, C = \frac{\varepsilon o k A}{d}$$

Special Case 2:

Multiple dielectric in series of thickness $t_1, t_2, \ldots, t_n$

$$C = \frac{\varepsilon o A}{\frac{t_1}{k_1} + \frac{t_2}{k_2} + \cdots + \frac{t_n}{k_n}}$$



Special case 3: Multiple dielectric in parallel of thickness $t_1, t_2, \ldots, t_n$

$$C = C_1 + C_2 + \ldots + C_n = \frac{\varepsilon o}{d}\left[k_1 A_1 + k_2 A_2 + \cdots + k_n A_n\right]$$

If $A_1 = A_2 = \ldots = A_n$ = A/n then $C = \frac{\varepsilon o A}{nd}\left[k_1 + k_2 + \cdots + k_n\right]$

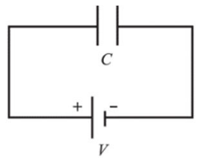
Special Case 4: If the space between the two plates is filled by conducting slab (k=∞)

$$Thus,\ C = \frac{\varepsilon o A}{d-t} = \frac{d}{d-t}Co\ where\ Co = \frac{\varepsilon o A}{d}$$

Energy Stored in capacitor:

Work done is stored in the form of electrostatic energy in the electric field between the plates.

Consider a capacitor of capacity C connected to a DC supply of V volts. Let q be the charge on the capacitor and voltage across it is V. Thus, C=q/V



Say we transfer a small charge dq between the plates. The work done for this would be

dW = V dq = q/C . dq

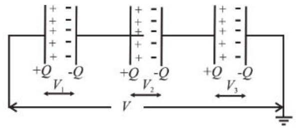
Total work done is

$$W = \int dw = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\int_0^Q q dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

$$Other\ forms.\ W = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Combination of Capacitance:

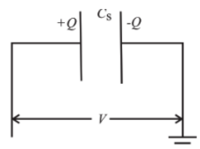
Series : The charge on the plates is same but Volatage is different



V = V1 + V2 + V3

$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

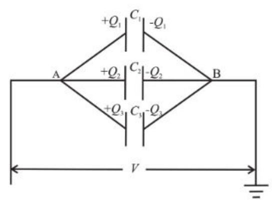
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Generalizing, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \ldots \ldots + \frac{1}{C_n}$

$$if, C_1 = C_2 = \cdots = C_n = C, then\ \frac{1}{C_{eq}} = \frac{n}{C}. thus, C_{eq} = \frac{C}{n}$$

Parallel : The capacitors have same potential difference but plate charges are different Q_1, Q_2, Q_3



Let total charge be Q

thus, $Q = Q_1 + Q_2 + Q_3$

$C_pV = C_1V + C_2V + C_3V$

Thus, $C_p = C_1 + C_2 + C_3$

Generalizing, $C_{eq} = C_1 + C_2 + \ldots\ldots\ldots + C_n$

if $C_1 = C_2 = \ldots = C_n = C$, then C_{eq} = nC

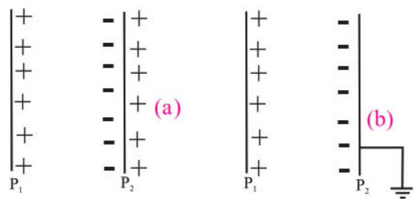
Principle of Capacitor:

Consider a positive plate P1 with charge Q and V_1 volts and capacity $C_o = Q/V_1$. When a neutral plate P2 is brought close to it, the inner side of P2 will induce a negative charge and positive charge will reside on the outside of P2. On earthing P2, the positive charge of P2 will get neutralized hence the potential of P2 decreases (say the value is V_2). Now the potential difference between the two plates is $V_1 – V_2$.

Thus,

$$C = \frac{Q}{V_1 - V_2}$$

Clearly C > Co. Thus, capacity of plate P1 is increased by placing an identical earthed metal plate P2 near it.



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